The equations derived by regression apply only for the region in which they are determined. Therefore, we refer the equation to the zero point, at which the contents of the nonsignificant elements correspond to the mean levels, i.e., Cr 10; V 8.5; Ga 8.5; Ge 1.5, while the contents of the significant elements correspond to the lower limits. The reduced contents for tungsten and rhenium coincide with the actual ones, since the minimal contents of these elements in these alloys were zero. The molybdenum contents in the alloys varied from 4 to 6%, so the reduced content is $(\%\overline{Mo}) = (\%Mo) - 4\%$. Then the equation is

$$\Delta\lambda_p = -2.58 + 2.3(\%\overline{Mo}) + 1.57(\%\overline{W}) - 1.24(\%\overline{Re}).$$

The constant term in this equation is negative, i.e., λ_p for the zero alloy is raised after annealing, and by a substantial amount. This is due to submicroscopic regions with short-range order of Ni_3Ga type, where some of the Ga atoms are replaced by other alloying elements. There are increases in bond strength and in the proportion of covalent component due to these regions, which raise λ_p after annealing. Tungsten and molybdenum have metallic bonds and reduce the total proportion of covalent bonding and so reduce λ_p. The electron shell in Re is aspheric, so there is a certain proportion of covalent character in the Re bonds. The effects of Re when the K state arises are increased because it has an electronegativity high relative to that of nickel.

NOTATION

λ, thermal conductivity, W/m·K; λ_p, phonon thermal conductivity component, W/m·K; λ_{pt}, phonon thermal conductivity component after tempering and aging, W/m·K; $\Delta\lambda_{p\,ht}$, variation in phonon thermal conductivity component due to heat treatment, W/m·K; λ_e, electron thermal conductivity component, W/m·K; $\overline{Mo}$, $\overline{W}$, $\overline{Re}$, reduced contents of molybdenum, tungsten, and rhenium, respectively, %.

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COMBINED SOLUTION OF THE PROBLEM OF EXTERNAL HEAT TRANSFER AND HEAT TRANSFER INSIDE A GAS RADIATION TUBE FOR THERMAL FURNACES WITH A PROTECTIVE ATMOSPHERE

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A general mathematical model of external heat transfer and heat transfer inside radiation tubes for furnaces with protective atmospheres (conjugate problem) is developed. An algorithm for solving the conjugate problem is developed.

In engineering practice, broad use is made of thermal units which include two working volumes separated by a wall: in the first, external heat-transfer processes occur; the second, as a rule, is in the form of a cylinder, in which there occurs the motion of dusty gases (metallic recuperators), petroleum products (tubular furnaces of the petroleum and

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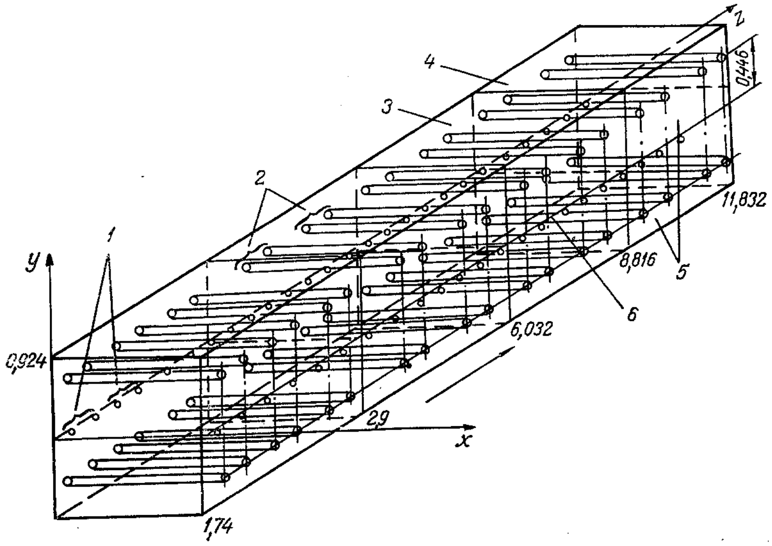

Fig. 1. Zonal model of furnace with radiation tubes (the arrow indicates the direction of metal motion): 1) roller zones; 2) radiation tubes; 3) side walls, 4) roof; 5) volume zones of protective gas; 6) metal; x, y, z, m.

gas industry), or a mixture of fuel gases and air (furnaces with radiation tubes). Heat-transfer processes in working volumes are closely interrelated, which means that it is necessary to create mathematical models of conjugate heat transfer in units of similar type.

The present work investigates a through roller furnace with a protective atmosphere equipped with gas radiation tubes. The methods of calculating external heat transfer in these furnaces [1-4] have several deficiencies: 1) the geometric scheme of the calculations is simplified (for example, the generatrix length of the cylindrical roller surfaces and radiation tubes is taken to be infinite); 2) no account is taken of the longitudinal radiant fluxes along the furnace channel; 3) the surfaces of the rollers and the furnace barriers are taken to be adiabatic; 4) experimental values of the radiation-tube efficiency are used to determine the thermal power of the furnace.

In the present work, the zonal method is applied for the conditions of the furnace working space [4]. An important stage in realizing the zonal-calculation method is the construction of a geometric model [5]. This stage includes the division of the furnace into individual zones and specification of the quantities determining the heat and mass transfer between the zones. The geometric model (Fig. 1) for the active furnace of the Pervoural'skii new-tube plant is divided into four theoretical sections over the length. In each section, four volume zones of protective gas, four of metal, four roof zones, four size walls, the two ends of the furnace, nine radiation tubes, and 13 rollers — altogether 40 zones — are separated. Metal divides the model into two heating zones over the furnace height: the upper ("top" of the furnace) and lower ("bottom" of the furnace) zones, between which there is radiant heat transfer. In connection with this, the angular coefficients of the radiation are determined separately for the "top" and "bottom" of the furnace.

The calculation is based on the system of heat-transfer and thermal-balance equations written for n surface zones [6-8]

$$\sum_{\substack{i=1\\i\neq j}}^{n-1} A_{ij}^{\Sigma}T_i^4 - A_{jj}^{\Sigma}T_j^4 + \sum_{\substack{i=1\\i\neq j}}^{n-1} q_{ij}T_i - q_{jj}T_j + Q_j = 0, \tag{1}$$

$$A_{ij}^{\Sigma} = \sigma_0 F_i \Sigma \bar{f}_{ij}^{\lambda} \bar{\varepsilon}_i^{\lambda} \bar{\alpha}_i^{\lambda}, \tag{2}$$

Analysis of the method of zonal calculation shows that it is expedient to choose an algorithm where the resolving angular coefficients f_{ij} are determined in two stages [9]. In the first stage, using a Monte Carlo program (the statistical-test method) [4], the geometric angular coefficients φ_{ij} are determined. In the second stage, the resolving angular coefficients f_{ij} are calculated, taking account of reemission of the energy from the

TABLE 1. Values of Variables and Coefficients for Eq. (4)

φ	a_φ	b_φ	c_φ	d_φ
ω/r	r^2	r^2	$\mu_э$	$-r\left[\frac{\partial}{\partial x}\left(\frac{u^2+v^2}{2}\right)\frac{\partial\rho}{\partial r}-\frac{\partial}{\partial r}\left(\frac{u^2+v^2}{2}\right)\frac{\partial\rho}{\partial x}\right]$
Ψ	0	$1/\rho r^2$	1	$-\omega/r$
f	1	Γ_f	1	0
h	1	Γ_h	1	$\frac{\mathrm{Bu}}{\mathrm{Bo}}(2\nu^4-I_r-J_r)$

boundary surfaces, from the solution of a system of linear equations [10]

$$f_{ij}=\varphi_{ij}+\sum_{k=1}^{n}R_k\varphi_{ik}f_{kj}. \tag{3}$$

The internal heat liberation Q_j in Eq. (1) is now considered in more detail for the radiation-tube zone. Earlier [4], Q_j was determined as the product of the radiation-tube efficiency and the potential thermal power supplied. At the same time, the efficiency depends on the thermal power itself, the air flow-rate coefficient, the heating temperature of the air, and other factors which may vary in the course of radiation-tube operation. Data regarding the dependence of the efficiency on these factors is not available in the literature as yet. In connection with this, it is of interest to investigate theoretically the heat and mass transfer in a radiation tube by the combined solution of hydrodynamic, thermal, and diffusional problems in the combustion of gaseous fuel.

The appropriate system of differential equations describing the hydrodynamics, mixing, and heat transfer for a flow with recirculation is written in the form of a single equation, as in [11, 12]

$$a_\varphi\left[\frac{\partial}{\partial x}\left(\varphi\frac{\partial\Psi}{\partial r}\right)-\frac{\partial}{\partial r}\left(\varphi\frac{\partial\Psi}{\partial x}\right)\right]-\frac{\partial}{\partial x}\left[b_\varphi r\frac{\partial(c_\varphi\varphi)}{\partial x}\right]-\frac{\partial}{\partial r}\left[b_\varphi r\frac{\partial(c_\varphi\varphi)}{\partial r}\right]+rd_\varphi=0. \tag{4}$$

The values of the variables and coefficients are given in Table 1.

The density of the gases is determined from the equation of state of an ideal gas, taking account of the mixture composition. In calculating the specific heat of the mixture, its composition is also taken into account in calculating the specific heat of the mixture, but the specific heat of an individual component is taken to be independent of the temperature.

Combustion is considered on the basis of a simple chemical reaction between two gases with the formation of a third (fuel + oxidant = combustion products). The reaction rate overall is determined by the mixing of the materials. The influence of turbulent concentration pulsations on the reaction rate is taken into account [13].

The radiation for absorbing and emitting media is calculated from a model with two fluxes in the radial direction, analogous to that in [12]. The equations for the radiant fluxes are as follows

$$\frac{d(rI_r)}{dr}=r\left(-\mathrm{Bu}I_r+\frac{J_r}{r}+\mathrm{Bu}\,\nu^4\right),$$
$$\frac{d}{dr}(rJ_r)=r\left(\mathrm{Bu}\,J_r+\frac{J_r}{r}-\mathrm{Bu}\,\nu^4\right). \tag{5}$$

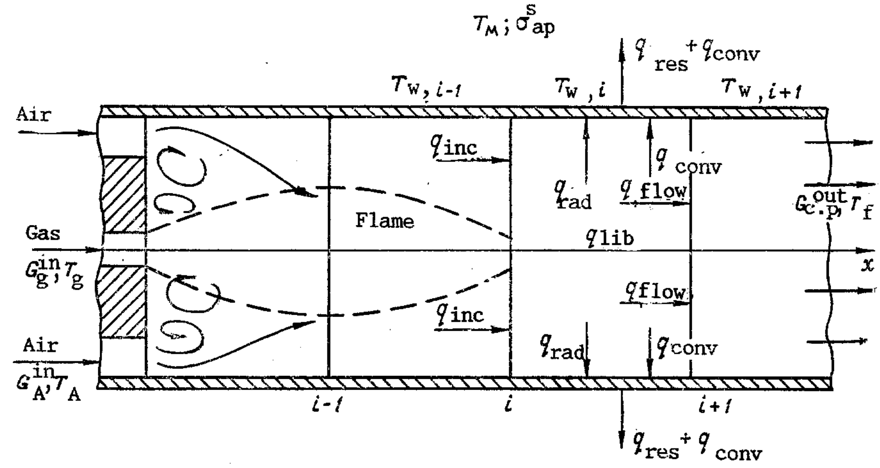

Fig. 2. One-dimensional radiation-tube scheme; the arrows indicate the direction of flow motion. T, °K; σ^s_{ap}, W/m^2·K^4; q, W/m^2; G, kg/sec.

The solution of Eq. (4) is obtained on the basis of a numerical finite-difference iterative method of Gauss—Seidal type using upper and lower relaxation [11]. The convective terms of Eq. (4) represent a "counterflow" scheme. In calculating the radiant fluxes, finite-difference analogs of the preliminarily obtained analytical solutions of Eq. (5) are used.

The corresponding physical problem is illustrated in Fig. 2. Fuel gas enters the radiation tube at the end and moves along the center, while air moves along the periphery; as these two components mix, combustion occurs (arrows indicate the direction of flow). The following boundary conditions are specified to solve the problem: at the inlet to the outer annular slot, the velocity profile of the air is uniform, while in the central hole the velocity profile of the fuel gas is uniform; at the side wall and the wall in the inlet cross section, the adhesion condition is assumed (v = u = 0); at the axis, the symmetry condition for all the variables is assumed; at the outlet, a stabilized profile of the combustion products is assumed. At the side wall, the temperature, emissivity, and impermeability condition — i.e., $\partial f/\partial r = 0$ — are specified. The temperature in the inlet cross section is taken to be equal to the air temperature at the inlet. The distribution of the turbulent kinematic viscosity in the tube cross sections is specified so as to agree with stabilized gas flow in a smooth tube [14]. The transfer coefficients for the concentration Γ_f and total enthalpy Γ_h are calculated from the formulas $\Gamma_f = \mu_e/Pr_f$, $\Gamma_h = \mu_e/Pr_h$, where $Pr_f = Pr_h = 0.9$, as in [15].

Calculations of the heat and mass transfer in a RNU-200 radiation tube have been performed by the above method. The resulting formula obtained for determining the fuel-burnup coefficient κ along the length of the tube is

$$\varkappa = 1 - \exp\left[-4\,\frac{x}{D}\,\mathrm{Re}^{-0,3}\right]. \tag{6}$$

Note that calculation of κ according to Eq. (6) corresponds to the experimental data of the All-Union Scientific-Research Institute of Metallurgical Heat Engineering on fuel burnup in a radiation tube. To estimate the influence of nonuniformities in the temperature and velocity fields on the radiation and convective heat transfer, the actual heat fluxes q^a_c and q^a_r obtained by the present method are compared with those calculated from the formulas for the one-dimensional problems

$$q^0_r = \sigma^{int}_{ap}(T^4_f - T^4_w),$$
$$q^0_c = \alpha_\infty (T_f - T_w). \tag{7}$$

The coefficients $k_r = q^a_r/q^0_r$, $k_c = q^a_c/q^0_c$ characterizing the difference between the actual local heat flux at the wall and that calculated from Eq. (7) are determined.

Analysis of the heat transfer inside the radiation tubes shows that there are two characteristic sections over the length of the tube: from the beginning and to the point where $T_w = T_f$, the section in which $T_w > T_f$; and from this point to the end of the tube, the section where $T_f > T_w$. The main nonuniformity coefficient $k_{r,i}$ and $k_{c,i}$ are calculated, together with the mean Nusselt-number ratio for radiant and stabilized flow Nu_r/Nu_∞ for these characteristic sections and for the tube as a whole, according to the formulas

$$k_{r,i} = \frac{\int_{x_1}^{x_2} q_r^a dx}{\int_{x_1}^{x_2} q_r^0 dx}, \quad k_{\kappa,i} = \frac{\int_{x_1}^{x_2} q_c^a dx}{\int_{x_1}^{x_2} q_c^0 dx}, \quad \frac{Nu_r}{Nu_\infty} = \frac{\int_{x_1}^{x_2} q_r^a dx}{\int_{x_1}^{x_2} q_c^0 dx}. \tag{8}$$

The use of the given method in the zonal calculation of through roller furnaces is practically impossible, because of the large amount of machine time required. In connection with this, it is proposed to consider heat transfer in a radiation tube by a one-dimensional scheme, calculating the burnup coefficients from Eq. (6) and taking account of the influence of nonuniformities in the temperature and velocity fields over the tube radius on the heat transfer in terms of k_c and k_r. Then the heat-transfer equation in the radiation tube with an air flow-rate coefficient α larger than unity (in the radiation tube $\alpha > 1$) is written in differential form as follows

$$\begin{gathered}
(G_{c.p}c_{c.p} + G_A c_A + G_g c_g)\frac{dT_f}{dx} = -\frac{dG_f}{dx}Q_n^r + \\
+ \alpha_\infty k_c (T_w - T_f)\pi D + \sigma_{ap}^{int} k_r (T_w^4 - T_w^4)\pi D, \\
G_g = G_g^{in}(1-\varkappa), \\
G_A = G_A^{in}\left(1 - \frac{\varkappa}{\alpha}\right), \\
G_{c,p} = G_g^{in} + G_A^{in} - G_g - G_A.
\end{gathered} \tag{9}$$

The wall temperature of the radiation tube varies over the length and is determined from the heat-transfer conditions of the tube with the working space of the furnace as follows

$$(T_f - T_w)\alpha_\infty k_c + \sigma_{ap}^{int}(T_f^4 - T_w^4)k_r = (T_w^4 - T_M^4)\sigma_{ap}^s + q_{conv}. \tag{10}$$

The flow temperature at the inlet to the tube is found, taking account of the heating of the air supplied to the radiation tube in the recuperator containing spent dusty gases.

Using Eqs. (9) and (10), the temperature distribution in the flow at the wall and the heat-flux distribution over the length of the radiation tube may be determined. To calculate the heat transfer inside the radiation tube, Eqs. (9) and (10) are reduced to finite-difference form, dividing the tube into a certain number of sections over the length. The apparent radiation coefficient for each section is determined using the two-band selectivity model [16].

The method here developed has been used to calculate the heat transfer in a RNU-120 radiation tube in comparison with the experimental data of the All-Union Scientific-Research Institute of Metallurgical Heat Engineering (IMHE).* The theoretical and experimental temperature distributions for the flow and the radiation-tube wall (Fig. 3) are in sufficiently good agreement. In Fig. 3, $T_f > T_w$, since there are no experimental data for the section with $T_f < T_w$, and no theoretical results are given here either.

The given method of calculating the heat transfer in a radiation tube is used in solving the conjugate heat-transfer problem in a furnace with radiation tubes (Fig. 4) as follows. First, the resulting heat fluxes for the radiation tubes are specified and the problem of external heat transfer is solved in the furnace and, as a result, the resulting fluxes are again found (stage I). Then, the method of [16] is used to determine the "apparent" radiation coefficient for the radiation tubes according to the following formula (stage II)

*The experiments were conducted by V. I. Maslov and O. N. Bondarenko.

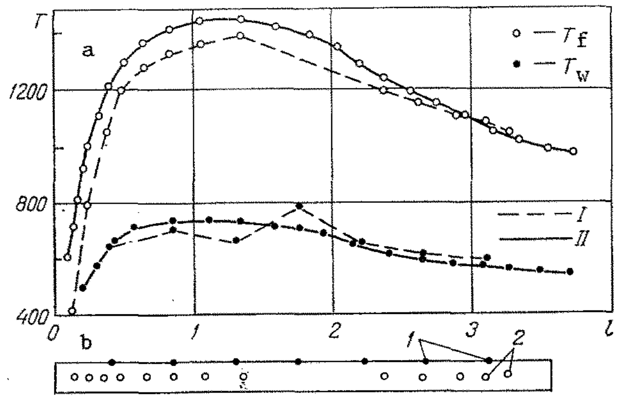

Fig. 3. Temperature distribution in the flux (T_f, °C) and at the wall (T_w, °C) over the length l, m, of a radiation tube (a) and distribution of measuring holes and thermocouples (b): I) experimental data obtained on the IMHE stand; II) calculation by an engineering method; 1) thermoucouples; 2) measuring holes.

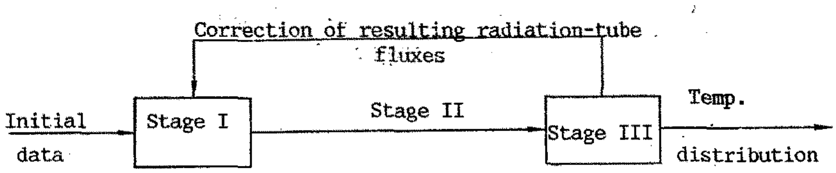

Fig. 4. Block diagram of the algorithm for solving the conjugate heat-transfer problem in a furnace with radiation tubes.

$$\sigma_{ap}^{s} = \frac{q_{res}}{T_м^4 - T_w^4}.$$

Next, a thermal calculation of the radiation tube is undertaken by the one-dimensional method, and, as a result, the heat flux from the tube into the furnace space and the mean wall temperature of the tube are determined (stage III). The calculations are repeated until the fluxes from the tube coincide with specified accuracy in two successive iterations.

It should be added here that the use of a two-dimensional method based on Eq. (4) and the one-dimensional method in Eqs. (9) and (10) in the general procedure for calculating the heat transfer in a furnace with radiation tubes is of independent importance for the calculation and analysis of the thermal work of heat-transfer apparatus in metallurgical furnaces, tubular furnaces of the petroleum and gas industry, etc.

NOTATION

A_{ij}^{Σ}, selective radiation-transfer coefficients for surface zones; $\bar{f}_{ij}^{\lambda}$, mean (within the limits of the radiation band) reduced resolving angular radiation coefficient from zone i to zone j; u, v, axial and radial velocity components; ρ, density of medium; ρ_m, u_m, mean density and velocity of the medium; D, internal diameter of tube; Bu, Bo, Bouguer and Boltzmann numbers; ν, relative temperature, $\nu = T/T_m$; T, T_m local and mean (over the tube volume) temperatures: I_r, J_r, radiant fluxes in the direction of the radial coordinate and in the opposite direction, referred to $\sigma_0 T_m^4$; σ_0, emissivity of absolutely blackbody; σ_{ap}^{int}, "apparent" emissivity for the internal space of the radiation tube; T_f, mean flow temperature of the gas in the cross section; T_w, wall temperature of radiation tube; α_∞, convective heat-transfer coefficient with stabilized flow in tube; κ, degree of fuel burnup; Re, Reynolds number; x, longitudinal coordinate; k_c, i, coefficient characterizing the influence of nonuniformity in the temperature and velocity field on the convective heat transfer; $k_{r,i}$, the same for radiant heat transfer; G_g, G_A, $G_{c.p}$, current mass flow rates of gas, air, and combustion products; G_g^{in}, G_A^{in}, mass flow rates of air and gas at the inlet to the radiation tube; c_g, c_A, $c_{c.p}$, specific heat of gas, air, and combustion products; σ_{ap}^{s}, "apparent" emissivity for the system consisting of the radiation tube and the working space of the furnace; T_M, temperature of metal.

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